Buckling Analysis in 3D Warp

In the Autodesk Moldflow Insight 2013 release, only small and large deflection analyses are provided in the three-dimensional (3D) Warp simulation of polymer injection molded components. In the Autodesk Simulation Moldflow Insight 2014 release, a buckling feature for three-dimensional warpage simulation of injection molded components has been developed. This feature uses a new fast parallel eigen-solver, which combines the AMG-CG equation solver, with the subspace eigenvalue iteration algorithm. The buckling analysis has also been extended to the insert over-molding and 2-shot over-molding processes. The presented method has provided an efficient tool for predicting if an injection molded part buckles after ejection from the mold.
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Introduction

As typical polymer injection molded parts have two distinctive properties: low Young's moduli of plastics (that are often two orders of magnitude less than steel), and thin-walled in nature, buckling may occur during the injection molding process. The molded part may buckle on ejection because of the in-mold residual stresses, which are dependent on the time, temperature and pressure processing history of the entire part. These stresses have to be rebalanced, causing the buckling. Buckling analysis is useful for optimizing part and mold design, material choice, and processing parameters to help control part deformation. Classical linear buckling analyses generally yield sufficiently accurate results for most practical injection molded problems, in which only small displacements and deformations occur before an instability point is reached.

The 3D flow, cooling and warpage simulation technique was developed several years ago [1, 2, 3]. The high gradient variation of velocity and temperature in gap-wise directions, and the limitation of maximum element aspect ratio, necessitate the use of many elements across the part thickness. Consequently, the number of elements for a 3D analysis has becomes very large. The issue becomes worse in the warpage simulation as the tetrahedral elements must be upgraded from first-order elements to second-order elements to get reliable warpage simulation results for thin-walled injection molded parts. Although the number of nodes per element is increased only from 4 to 10, the total number of nodes in a 3D mesh will be increased by around 6 times. This leads to an excessive memory requirement and computation time, particularly when geometric nonlinearity is included in the warpage analysis [4]. To overcome this, a very fast and robust parallelized AMG-CG equation solver is used whose convergence rate is approximately independent of mesh resolution [5].

A buckling analysis in the warpage simulation is essentially to solve a generalized eigen-problem. The traditional direct-solver-based eigen-solvers, which are widely used in the commercial finite element packages, become prohibitively expensive for large-scale finite element models of injection molding simulation. In this paper, a new fast parallel eigen-solver has been developed, which is combined with the AMG-CG equation solver and the subspace eigenvalue iteration algorithm. With the fast generalized eigen-solver and the rapidly increasing availability of computing power, buckling prediction in the 3D warpage simulation has become a realistic option.

The buckling analysis has also been extended to the injection insert over-molding process and two-shot over-molding process. The buckling analysis results for two injection simulation cases are presented.

Buckling Algorithm

The aim of a buckling analysis is to find one or more of the lowest positive eigenvalues and corresponding eigenvectors for the following generalized eigen-system.

\[
(K^t + \lambda K^s) U = 0
\]

Where \(K^t\) is the linear stiffness matrix, \(K^s\) is the geometric or initial stress matrix. \(\lambda\) and \(U\) are the eigenvalue and corresponding eigenvector respectively.

The Lanczos method and subspace iteration method (or variants of these two iterative schemes) are very widely used for the solution of large eigenvalue problems in finite
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The subspace iteration method is chosen as it is a particularly effective algorithm in finding a few lowest eigenvalues and their corresponding eigenvectors for a large generalized eigen-problem. Both inverse iteration and the generalized Jacobi iteration methods have been incorporated into the subspace iteration algorithm. When using parallel processing in a finite element analysis, in shared memory, and distributed memory processing modes, the subspace iteration method allows the parallel solution of multiple iteration vectors, which can result in a large computational benefit. The Lanczos method (working on individual vectors) intrinsically is disadvantaged by parallelization.

The subspace iteration algorithm starts with the initial matrix $X_1$ for the requested eigenvectors. It projects the large eigenvalue system into a much smaller subspace. The main iteration steps include:

1. Obtain $X_{k+1}$ by solving a set of linear equations:
   $$K^l X_{k+1} + K^o X_k = 0$$

2. Find the reduced linear stiffness matrix and initial stress matrix,
   $$K^l_{k+1} = X^T_{k+1} K^l X_{k+1}$$
   $$K^o_{k+1} = X^T_{k+1} K^o X_{k+1}$$

3. Use the generalized Jacobi iteration method to solve the eigen-equation of the projected matrices for its eigenvalues $A_{k+1}$ and eigenvectors $Q_{k+1}$
   $$K^l_{k+1} Q_{k+1} + K^o_{k+1} A_{k+1} = 0$$

4. Find an improved approximation of the eigenvectors in the original large eigenvalue system.
   $$X_{k+1} = X_{k+1} Q_{k+1}$$

The most time-consuming linear equation solution step is traditionally solved using the direct solver. Its distinct advantage is that the repeated linear equation solution during the subspace iteration is just a backward substitution once the sparse direct factorization of $K^l$ is available. However, as we discussed in the introduction, the number of unknowns tend to be very large in the 3D injection molding warpage simulation. In some real-world industrial applications, the number of unknowns can reach several millions or even tens of millions. Therefore, performing the sparse direct factorizations could become prohibitively expensive. The excessive memory requirement and extreme computational time for the large-scale eigen-system makes the direct solver an unrealistic option even on modern powerful workstations. A feasible approach is to replace the sparse direct method with an AMG-CG equation solver within the subspace iteration algorithm.

As $K^l$ remains unchanged in the solution process except that the shifting procedure is applied, the preconditioning calculation in AMG-CG is done once only during the entire subspace iteration. On the other hand, on modern cache-based machines, data locality is often the most critical factor affecting performance. The linear equation solution of multiple
iteration vectors in step 1 of the subspace iteration method enables efficient parallelization implementation by exploiting the data locality [7].

The negative eigenvalue has no physical meaning in the warpage analysis. If negative eigenvalues occur, the shifting procedure is automatically activated for getting the lowest positive eigenvalues.

**Buckling for Injection Over-molded parts**

Injection over-molding has become a popular fabrication process in recent years. Many industries use over molding to produce a wide range of parts. 3D simulation for the injection over-molding process was developed. A multi-point constraint (MPC) approach is employed for handling the cavity-insert interface in the warpage solution [3]. The analysis starts by identifying the interface area between the insert mesh and the cavity mesh. It assumes that the insert nodes at the interface are the master nodes, and the cavity the slave nodes. It is recommended that the density of the insert mesh be coarser than that of the cavity mesh. Based on the relative geometric position and the displacement interpolation function of the element, the MPC equations between the degrees of freedom at the slave node and at the master nodes can be established.

\[ U_s = BU_m \]

Where \( U_s \) is the slave displacement vector, and \( U_m \) is the master displacement vector. \( B \) is the master-slave displacement relationship matrix. With these constraints the structural performance is expected to be identical to the composite structure with perfect bonding between inserts and cavities.

The elimination method based on the Lagrange multiplier formulation is used for handling MPC equations, in which the constrained degrees of freedom and multipliers are eliminated, thereby yielding lower order matrix equations for unconstrained degrees of freedom [8]. The buckling load and corresponding buckling modes for injection over-molded parts can be obtained by solving the following generalized eigen-problem.

\[
(K_{mm} + (B^T K_{sm} + K_{ms} B) + B^T K_{ss} B) U_m + \\
\lambda (K^\sigma_{mm} + (B^T K^\sigma_{sm} + K^\sigma_{ms} B) + B^T K^\sigma_{ss} B) U_m = 0
\]

Where \( K_{mm}, K_{ss}, K_{sm} \) and \( K_{ms} \) are the master zone, slave zone and coupling zones of the linear stiffness matrices respectively. \( K^\sigma_{mm}, K^\sigma_{ss}, K^\sigma_{sm} \) and \( K^\sigma_{ms} \) are the master zone, slave zone and coupling zones of the initial stress matrices respectively.
Graphical User Interface Design for Large Deflection Analysis

In the Autodesk Moldflow Insight 2012 release, large deflection analysis is available for 3D Warp analyses which do not use the mesh aggregation option. The Buckling option is found in the 3D Warp Advanced options dialog, when the option to Use mesh aggregation and 2nd-order tetrahedral elements is not selected on the Warp Settings page of the Process Settings Wizard.

Figure 1: Buckling Analysis Solver Parameters dialog

There are two choices in the Solver parameters dialog: Number of eigenvalues to output and Convergence tolerance for eigenvalue calculation. The default settings are normally good enough. However, you can output more eigenvalues to see several buckling load and buckling modes. In some special cases, the lowest positive eigenvalues may be close to each other.

Numerical Examples and Discussion

The buckling feature in 3D warpage simulation has been with built-in pre-and-post processing capabilities. Initial strain conditions are derived from a prior flow analysis of the injection molding process without any user intervention required.

The implementation has been verified by comparing the predicted buckling load levels and buckling shapes with the analytical solutions for several simple thin plates under different external loadings. Figure 2 shows the 3D buckling analysis results of a simply supported plate subjected to an in-plane compressive loading. Both the predicted buckling load level and buckling shape are consistent with the analytical solution [9].
The simulation results of two injection molded parts are presented to validate the algorithms. The first model is a tray model, as shown in Figure 3. This is a very thin-walled part with an average wall thickness of 0.80 mm. Although a midplane shell solution would be the best option for this uniform wall thickness geometry, it is analyzed using a 3D simulation to validate the solution. 169,017 4-node tetrahedral elements are used in the flow simulation, and all elements have been upgraded to 10-node tetrahedral elements for the warpage analysis. The number of nodes used in the flow and warpage analysis was 31,047 and 236,421 respectively. The second-order tetrahedral element is used for avoiding the notorious locking issues in the first-order tetrahedral element [8].

The buckling analysis predicts a buckling load factor of 0.9013. This means that the part buckles at 90.13 per cent of the in-mold residual stresses. Figure 5 shows the buckling mode, which is consistent with the experimental results in Figure 4. To get the final post-buckling shape, a large deflection analysis is required.

The second case is an over-molded mobile phone cover, as shown in Figure 6. Since the insert has much less shrinkage than its surrounding injected polymer, it is forced to bend upwards slightly. The buckling analysis predicts the lowest eigenvalue of 7.647, and the corresponding buckling mode is shown in Figure 7. It indicates that the part will not buckle after ejection from the mold.
It is worth mentioning that the 3D buckling solution developed in this paper has been successfully used for warpage simulation of large-scale finite element models of real-world injection molded parts. With the current approach, we solved the buckling problem of a large finite element model of over 6.5-million degrees of freedom on a HP Z800 workstation within 5 hours.

**Summary**

A buckling feature for 3D warpage simulation of injection molded components has been developed using a new fast parallel eigen-solver which combines the AMG-CG equation solver with the subspace eigenvalue iteration algorithm. The buckling analysis has also been extended to the insert over-molding and 2-shot over-molding processes. The presented method has provided an efficient tool for predicting if the injection molded parts buckle after ejection from the molds.

**References**


5. Z. Fan, C. Iwamura , S. Xu, F. Costa, Y. Supramaniam, Reducing the Computational Time for Three-Dimensional Simulation of Warpage of Injection Molded Plastic

